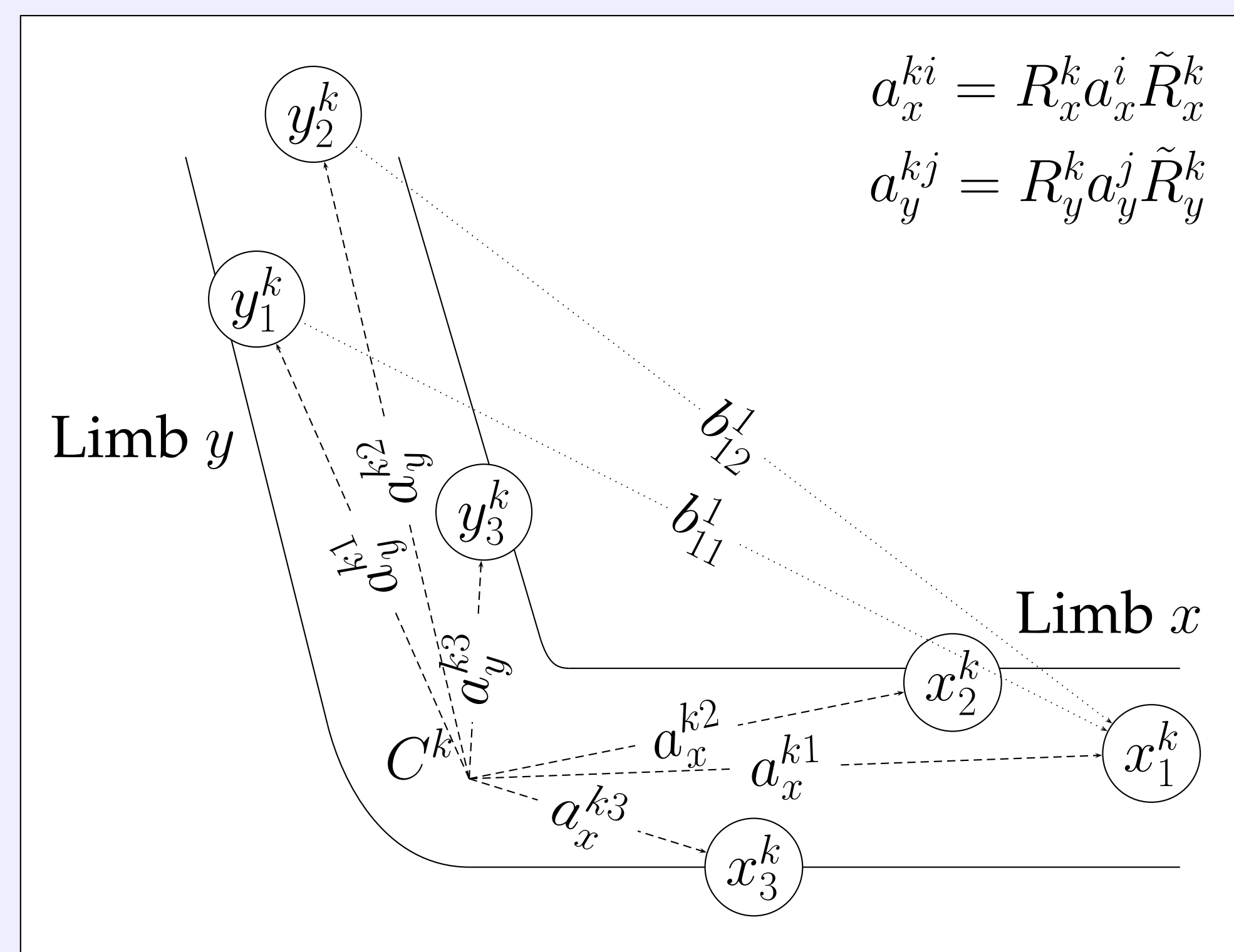


A Real-Time Sequential Algorithm for Human Joint Localization

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We present a novel approach to locating the Centres of Rotation (CoRs) of joints of a human skeleton from position only data. Unlike previous algorithms [3, 2], this algorithm can be implemented in a sequential fashion and as such the computational cost of updating the Centres of Rotation is independent of the number of data points previously available. The method is closed form, enabling real-time implementation.



1 Problem Statement

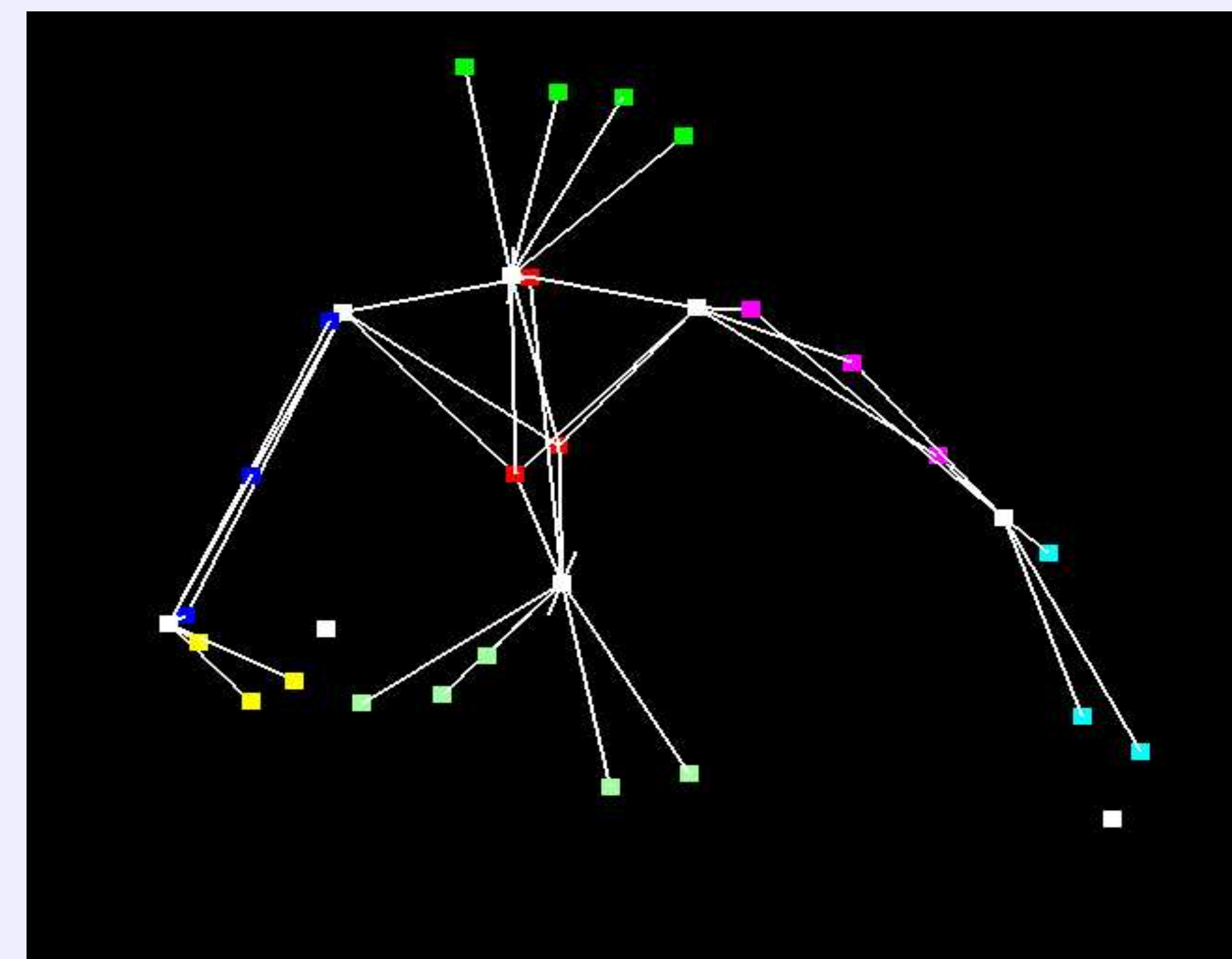
Find the location of the Centres of Rotation (CoRs) for a simple ball and socket model of a human skeleton using position only data obtained from optical motion capture.

This problem is of interest to many people working in the fields of Computer Graphics and Medical Diagnosis where automated, accurate skeleton generation would greatly reduce the amount of time required to setup a motion capture session by removing the need to place markers in precise locations.

Here we ignore the associated problems of clustering and graph pruning so as to concentrate on that of locating the CoRs.

2 Use of Geometric Algebra

The mathematical framework used in this poster is that of Geometric Algebra. The only real difference (as used here) from normal vector algebra lies in the use of Rotors R and their reverse $\tilde{R}$, to represent rotations. For an introduction to Geometric Algebra the reader is referred to [1].



3 The Cost Function

To find the optimal location of the CoR w. r. t. the marker positions, we need to define a cost function with a minimum at the true location of the CoR.

Let the Centre of Rotation for a pair of bodies, linked by a ball and socket type joint, in frame k be C^k . Let the vectors from this point to the position of each marker in the first frame, be denoted a_x^i for the body x and a_y^j for the body y where i and j are simply used to differentiate between markers.

Now consider the position of these markers in frame k ,

$$\begin{aligned} x_i^k &= C^k + R_x^k a_x^i \tilde{R}_x^k \\ y_j^k &= C^k + R_y^k a_y^j \tilde{R}_y^k \end{aligned}$$

Let b_{ij}^k be the vector from y_j^k to x_i^k . That is

$$\begin{aligned} b_{ij}^k &= x_i^k - y_j^k \\ &= R_x^k a_x^i \tilde{R}_x^k - R_y^k a_y^j \tilde{R}_y^k \end{aligned}$$

Now we can construct a cost function S ,

$$S = \sum_{k=1}^m \sum_{i=1}^{n_x} \sum_{j=1}^{n_y} [b_{ij}^k - (R_x^k a_x^i \tilde{R}_x^k - R_y^k a_y^j \tilde{R}_y^k)]^2$$

4 Finding The Optimal Solution

Given at least 3 markers on each limb R_x^k and R_y^k can be reconstructed in a least-squared sense using a geometric algebra version of Procrustes Method.

Now we differentiate w.r.t. the a_x^i and a_y^j vectors using the standard results $a \cdot Rb\tilde{R} = \tilde{R}aR \cdot b$ and $\partial_a(a \cdot b) = b$.

$$\begin{aligned} \partial_{a_x^i} S &= \sum_{k=1}^m \sum_{j=1}^{n_y} \{ -2\tilde{R}_x^k b_{ij}^k R_x^k + a_x^i - \tilde{R}_x^k R_y^k a_y^j \tilde{R}_y^k R_x^k \} \\ &= 0 \end{aligned}$$

rearranging

$$a_x^i = \frac{1}{m} \sum_{k=1}^m \tilde{R}_x^k \bar{b}^{ik} R_x^k + \frac{1}{m} \sum_{k=1}^m \tilde{R}_x^k R_y^k a_y^j \tilde{R}_y^k R_x^k$$

Similarly by differentiating w.r.t. a_y^j

$$a_y^j = \frac{1}{m} \sum_{k=1}^m -\tilde{R}_y^k \bar{b}^{jk} R_y^k + \frac{1}{m} \sum_{k=1}^m \tilde{R}_y^k R_x^k \bar{a}_x^i \tilde{R}_x^k R_y^k$$

where

$$\begin{aligned} \bar{b}^{ik} &= \frac{1}{n_y} \sum_{j=1}^{n_y} b_{ij}^k & \bar{b}^{jk} &= \frac{1}{n_x} \sum_{i=1}^{n_x} b_{ij}^k \\ \bar{a}_x^i &= \frac{1}{n_x} \sum_{i=1}^{n_x} a_x^i & \bar{a}_y^j &= \frac{1}{n_y} \sum_{j=1}^{n_y} a_y^j \end{aligned}$$

This pair of linear equations in the vectors a_x^i , a_y^j is sufficient to determine them by inverting a $3(n_x + n_y) \times 3(n_x + n_y)$ matrix. These vectors, in combination with the rotors already found, provide a number of estimates of the CoR in any frame. These are then averaged to provide a single answer.

The size of this matrix can be reduced to 6×6 by performing the average prior to solving for the vectors. Thus the computational cost of the algorithm is linear with number of frames and markers, and sequential updating of the parameters is possible.

5 Conclusions and Further Work

The algorithm as set out here works very well in practice and can run in real time with data from a Phasespace[4] motion capture system and will be demonstrated with a variety of datasets.

The fully sequential algorithm is limited to cases where at least 3 markers are available on both sides of the CoR. However, there is sufficient information to find the CoR and the rotation of both sections with only 2 markers per side. An iterative approach alternating between finding the optimal vectors for a given set of rotation angles in the ambiguous direction and finding the optimal angles for a fixed set of vectors, has been shown to work well. This optimization can be time consuming and so a real time implementation is not currently feasible.

References

- [1] C. Doran and A. Lasenby. *Geometric Algebra for Physicists*. CUP, 2003.
- [2] Sahan S. Hiniduma Udugama Gamage and Joan Lasenby. New least squares solutions for estimating the average centre of rotation and the axis of rotation. *Journal of Biomechanics*, 35:87–93, 2002.
- [3] M. Silaghi, R. Plänkers, R. Boulic, P. Fua, and D. Thalmann. Local and global skeleton fitting techniques for optical motion capture. In *Proceedings CapTech*. IFIP, Springer Verlag Heidelberg, 1998.
- [4] PHASESPACE, www.phasespace.com also found in the guerilla studio and on the AMD booth of the main exhibition at SIGGRAPH 2005.